R^2 and R^2_{adj}

Tests 1 and 2 can be used to evaluate the model's adequacy.

If I do not reject $\beta_j = 0$ for some j, I can remove that covariate

If I do not reject $\beta_1 = \dots = \beta_p = 0$, the whole model is useless.

How do I choose between different models?

- I can compare R^2 (larger R^2 means more variability explained)

However, if I use R^2 to compare nested models (i.e. one can be obtained starting from the other by fixing some parameters $= 0$), R^2 is not a valid measure.

consider (a) $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$

(b) $\tilde{y}_i = \tilde{\beta}_1 + \tilde{\beta}_2 x_i + \tilde{\beta}_3 w_i$ $\rightarrow$ I add one covariate

$R^2_{(a)} \leq R^2_{(b)}$ by construction: the SSR of model (b) can not be smaller than $SSR_{(a)}$.

In the worst case (if w_i is really useless), I set $\tilde{\beta}_3 = 0$ and I obtain $SSR_{(a)}$.

The more variables I include in the model, the larger R^2 will be.

In general:

+ covariates: R^2 increases; less interpretable; overfit

− covariates: parsimony; interpretable

of course, we want few covariates, but not too few!

- ADJUSTED R^2 $\qquad R^2_{adj} = 1 - (1 - R^2) \cdot \dfrac{n-1}{n-p}$

it is "adjusted" for the model dim. p

penalizes models with many covariates.

when I introduce a new covariate:

- R^2 can remain the same or increase
- R^2_{adj} can increase, remain the same, or decrease

$\Downarrow$

R^2_{adj} can be < 0!