31 oct - LEC 7

GEOMETRIC INTERPRETATION OF THE TEST

Consider again the representation of the model in an n-dimensional space.
Here, the variables $(\underline{y}, \underline{x}_1, \dots, \underline{x}_p)$ are n-dimensional vectors, with coordinates the observations on the n units.
The covariates $(\underline{x}_1, \dots, \underline{x}_p)$ identify a subspace of dimension p, $C(X)$.
This subspace is defined by all linear combinations $\beta_1 \underline{x}_1 + \dots + \beta_p \underline{x}_p = X\underline{\beta}$.
The mean of $\underline{Y}$ is $\underline{\mu} = X\underline{\beta}$ $\Rightarrow$ the mean of $\underline{Y}$ belongs to $C(X)$.
The vector $\underline{y}$ in general will not belong to $C(X)$: indeed we have seen that $\hat{\underline{\mu}} = \hat{\underline{y}}$ is the orthogonal projection of $\underline{y}$ onto $C(X)$.

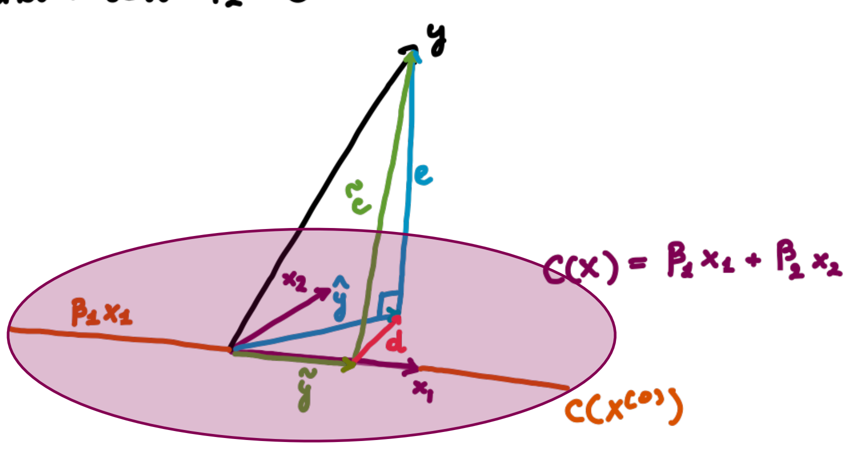
What happens when we compare NESTED models?
example with 2 variables $\underline{x}_1, \underline{x}_2$

Full model: $\underline{Y} = \beta_1 \underline{x}_1 + \beta_2 \underline{x}_2 + \underline{\varepsilon}$
$X = [\underline{x}_1 \ \ \underline{x}_2]$
$C(X)$ is the subset of $\mathbb{R}^3$ of all linear combinations $\beta_1 \underline{x}_1 + \beta_2 \underline{x}_2$ (dim = 2)
$\hat{\underline{y}} = \hat{\beta}_1 \underline{x}_1 + \hat{\beta}_2 \underline{x}_2$ is the orthogonal projection of $\underline{y}$ onto $C(X)$

Assume we want to test
$$\begin{cases} H_0: \beta_2 = 0 \\ H_1: \beta_2 \neq 0 \end{cases}$$

Under H_0, the reduced model is $\underline{Y} = \beta_1 \underline{x}_1 + \underline{\varepsilon}$
Here $X^{(0)} = [\underline{x}_1]$
$C(X^{(0)})$ is the subset of linear combinations $\beta_1 \underline{x}_1$ (dim = 1)
$C(X^{0})$ is defined by a straight line (and not the entire plane)
fitted values $\tilde{\underline{y}} = \tilde{\beta}_1 \underline{x}_1$; $\tilde{\underline{y}}$ belongs to $C(X^{(0)})$
→ This is a constrained estimate

example with 2 covariates x_1 and x_2
and 1 test $\beta_2 = 0$



$\hat{\underline{y}}$: projection on $C(X)$
$\tilde{\underline{y}}$: projection on $C(X^{(0)})$

The vector $\underline{d}$ is equal to $\hat{\underline{y}} - \tilde{\underline{y}}$ and also to $\tilde{\underline{e}} - \underline{e}$
Moreover $\underline{d} \perp \underline{e}$
$\Rightarrow$ Pythagoras thm. $\underline{e}^T\underline{e} + \underline{d}^T\underline{d} = \tilde{\underline{e}}^T\tilde{\underline{e}}$
$\Rightarrow$ $\underline{d}^T\underline{d} = \tilde{\underline{e}}^T\tilde{\underline{e}} - \underline{e}^T\underline{e}$

With the test about nested models, we are looking at the difference between the unconstrained estimate $\hat{\underline{y}}$ and the constrained one $\tilde{\underline{y}}$, or, equivalently, between the errors we commit under the unconstrained model ($\underline{e}$) and the restricted model ($\tilde{\underline{e}}$).