

Exercises: Simple Linear Regression

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October 18, 2024

(Thanks to Valentina Zangirolami for providing this material)

1 Mother and Daughter heights data

Let consider a sample of data with $n = 11$ observations (Table 1) with two variables:

- **mother's height** x (independent variable);
- **daughter's height** y (dependent variable).

Table 1: Mother and Daughter heights data: data are expressed in centimeters.

	1	2	3	4	5	6	7	8	9	10	11
x	153.7	156.7	173.5	157.0	161.8	140.7	179.8	150.9	154.4	162.3	166.6
y	163.1	159.5	169.4	158.0	164.3	150.0	170.3	158.9	161.5	160.8	160.6

We would like to find out if there exists a relationship between these two variables.

Exercise 1.1

Starting from the data (in Table 1), write the equation of the simple linear regression model. Compute \bar{x} , \bar{y} , $\sum_{i=1}^n x_i y_i$, $\sum_{i=1}^n x_i^2$ and, then, find the estimates of the linear model parameters.

Exercise 1.2

Given the results of the previous exercise, compute the fitted values for each i . Make a plot involving the observations of the couple (y, x) and the estimated regression line.

Exercise 1.3

Compute the residuals (e_i) and the unbiased estimate (S^2) of the variance σ^2 . Then, find

the estimates of the variances of \hat{B}_1 and \hat{B}_2 .

Exercise 1.4

Compute the total sum of squares (SST), the residual sum of squares (SSE) and the regression sum of squares (SSR). Then, find the coefficient of determination R^2 . Compute the correlation coefficient r_{xy} and its squared. What happens in this case?