Exercises: Simple Gaussian Linear Regression Part I

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October 18, 2024

(Thanks to Valentina Zangirolami for providing this material)
(The results obtained in the previous practical part - simple linear regression - can be useful)

1 Mother and Daughter heights data

Let consider a sample of data with $n = 11$ observations (Table 1) with two variables:

- **mother's height** x (independent variable);
- **daughter's height** y (dependent variable).

Table 1: Mother and Daughter heights data: data are expressed in centimeters.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
x	153.7	156.7	173.5	157.0	161.8	140.7	179.8	150.9	154.4	162.3	166.6
y	163.1	159.5	169.4	158.0	164.3	150.0	170.3	158.9	161.5	160.8	160.6

We would like to find out if there exists a relationship between these two variables.

Exercise 1.5
Starting from the data (in Table 1), write the equation of the gaussian simple linear regression model together with the associated assumptions. Explain the difference from simple linear regression (make a comparison between the assumptions of ex. 1.1 and this case).

Exercise 1.6
Let consider the following system of hypothesis

$$\begin{cases} \text{H0: } \beta_2 = 1 \\ \text{H1: } \beta_2 \neq 1 \end{cases}$$

Compute the t-test and the p-value.

Exercise 1.7 Let consider the following system of hypothesis

$$\begin{cases} \text{H0: } R^2 = 0 \\ \text{H1: } R^2 > 0 \end{cases}$$

Compute the F-test and the p-value. In this case, does an equivalent test exists? Specify the hypothesis, the test statistic and compute the p-value.

Exercise 1.8
Provide the confidence intervals for β_r, $r = 1, 2$ at level $1 - \alpha = 0.95$.

Exercise 1.9
During the theoretical lectures, you exploited the relationship between R^2 and the t-test to prove the equivalence among two statistical tests in the case of simple linear regression. Provide the formula and verify that it holds with the data. (In the exercise 1.4, you have already computed the R^2 and the components of deviance decomposition.)