EXERCISE 3 EXAM 3/9/2024

a) Y_i = number of breaks $i = 1, \dots, n$

$$x_{i1} = \begin{cases} 1 & \text{if material}_i = B \\ 0 & \text{if material}_i = A \end{cases}$$

$$x_{i2} = \begin{cases} 1 & \text{if tension}_i = M \\ 0 & \text{if tension}_i = \{L, H\} \end{cases}$$

$$x_{i3} = \begin{cases} 1 & \text{if tension}_i = H \\ 0 & \text{if tension}_i = \{L, M\} \end{cases}$$

model: $Y_i \sim \text{Poisson}(\mu_i)$ indep for $i = 1, \dots, 54$

$$\eta_i = \beta_1 + \beta_2 x_{i1} + \beta_3 x_{i2} + \beta_4 x_{i3}$$

$$\log(\mu_i) = \eta_i \iff \mu_i = e^{\eta_i}$$

b) consider two experiments with same tension and different material

exp. i : material A

exp. j : material B

$$\log \mu_i = \beta_1 + \cancel{\beta_2 \overset{0}{x_{i1}}} + \beta_3 x_{i2} + \beta_4 x_{i3}$$
$$\log \mu_j = \beta_1 + \beta_2 \overset{1}{x_{j1}} + \beta_3 x_{j2} + \beta_4 x_{j3}$$

$$\begin{pmatrix} x_{i2} = x_{j2} \\ x_{i3} = x_{j3} \end{pmatrix}$$

$$\Rightarrow \log \mu_j - \log \mu_i = \cancel{\beta_1} + \beta_2 + \cancel{\beta_3 x_{j2}} + \cancel{\beta_4 x_{j3}} - \cancel{\beta_1} - \cancel{\beta_3 x_{i2}} - \cancel{\beta_4 x_{i3}}$$
$$= \beta_2$$

β_2 is the difference in the log of the expected counts if I consider material B instead of material A, for fixed level of tension.

That is, the log of the expected number of breaks using material B decreases by 0.20 w.r.t. material A, for fixed level of tension.

c) $\begin{cases} H_0: \beta_2 = \beta_3 = \beta_4 = 0 \\ H_1: \text{at least one is} \neq 0 \end{cases}$

model B

$Y_i \sim \text{Pois}(\mu)$ $i = 1, \dots, n$

μ is common $\log(\mu) = \beta_1$

I use the LRT

$$W = 2[\hat{\ell}(\text{model A}) - \tilde{\ell}(\text{model B})] \overset{H_0}{\sim} \chi^2_3$$

since model B is the null model

$$W^{obs} = 2[\underbrace{\hat{\ell}(\text{model A}) - \tilde{\ell}(\text{saturated})}_{-D(\text{model A})} + \underbrace{\tilde{\ell}(\text{saturated}) - \tilde{\ell}(\text{null})}_{D(\text{null})}]$$

$$W^{obs} = D(\text{null}) - D(\text{model A}) = 297.37 - 210.39 = 86.98$$

Reject region is $R = (\chi^2_{3,1-\alpha}; +\infty)$

Since $W^{obs} > \chi^2_{3,1-\alpha}$ for all usual α, I reject H_0